

Pseudoscalar Quarkonium Exclusive Decays to Vector Meson Pair

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Abstract

The pseudoscalar quarkonia exclusive decays to light mesons still poses a challenge to the theoretical understanding of quarkonium properties in decay. In this work, we evaluate the processes of pseudoscalar heavy quarkonium decays into vector meson pairs, especially the helicity suppressed processes of $\eta_b \rightarrow J/\psi J/\psi$ and $\eta_c \rightarrow VV$. In the frame of NRQCD, the branching fraction of $Br[\eta_b \rightarrow J/\psi J/\psi]$ are evaluated at the next-to-leading order of perturbative QCD; and within the light-cone distribution formalism, we calculate also the higher twist effects in these processes. Numerical results show that the higher twist terms contribute more than what from the NLO QCD corrections in the process of $\eta_b \rightarrow J/\psi J/\psi$. It is found that the experimental results on $\eta_c \rightarrow VV$ are hard to be understood by merely the quark model and perturbative QCD calculation.

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I. INTRODUCTION

In high energy physics, heavy quarkonium study is one of the most interesting fields and it plays an important role in the understanding of the configurations of hadrons and the nonperturbative behavior of strong interaction. On one hand, the heavy quark masses enable the perturbative QCD(pQCD) calculation for quarkonium production and decay possible. On the other hand, due to the non-relativistic nature of heavy quarkonium, one may investigate their properties through a more transparent way, i.e. the effective theory of non-relativistic QCD(NRQCD) [1].

It is well known that the S-wave spin-triplet heavy quarkonium states, the J/ψ and Υ , can be produced directly in e^+e^- annihilation, and be measured via lepton pair decay mode distinctively. These characters lead to rich experimental data and deep investigations on them. While for their spin-singlet partners, the η_b and η_c , things are not that easy. At present, people know relatively much less about their properties, especially for η_b . For η_c , though there have been many measurements in experiment on its various decay modes, puzzles remain in confronting theoretical explanations to the experimental data, such as in η_c decay to vector meson pair [2, 3]. For η_b study, there have many theoretical scenarios been put forward [4–10], and several experiments been conducted [11–15]. However, it was fixed only in very recently by BaBar collaboration through $\Upsilon(3S) \rightarrow \eta_b + \gamma$ process [16] and later on confirmed by CLEO-c experiment [17]. About η_b so far we merely know the mass, its other properties are remaining unclear and waiting for further investigations. It is worth noting that both Babar and CLEO-c measurements are indirect ones. For further study on η_b physics, direct measurements on its decay products are necessary.

For the direct η_b detection, Braaten *et al.* suggested to measure its exclusive decay products, the J/ψ pair in η_b decays [6]. In comparison with the experimental result for $\eta_c \rightarrow \phi\phi$ and by some scaling assumptions, they estimated the branching ratio of $\eta_b \rightarrow J/\psi J/\psi$ mode to be $7 \times 10^{-4 \pm 1}$, which hence is promising to be observed in the Fermilab Tevatron Run II experiment. So far there has been no conclusive report from the experiment yet, and the theoretical estimation was questioned by Maltoni and Polosa [18]. In the expectation of helicity conservation rules [19], the leading order calculation in the nonrelativistic limit gives null result. The calculations on next-to-leading order QCD [7] and relativistic corrections

[8] both yield the branching ratios to be about 10^{-8} . Recently, Braguta *et al.* reevaluate the $\eta_b \rightarrow J/\psi J/\psi$ process in the light cone formalism and find that after including the next-to-leading twist contribution the branching fraction can be as large as $(6.2 \pm 3.5) \times 10^{-7}$ [9]. The authors of Ref. [9] claim that the result in [8] does not agree with theirs. The form factor obtained in Ref.[8] contains double logarithms, whereas they are absent in [9]. To carry on an independent calculation of the $\eta_b \rightarrow J/\psi J/\psi$ process is therefore one of the aims of this work.

Similarly, the processes $\eta_c \rightarrow VV$ are also governed by the helicity selection rules, but experiment gives extremely large results [2], which stands as a long term puzzle existing in the charmonium physics. The higher order radiative corrections give this issue no help, since they are all suppressed by the light quark masses, Although beyond the scope of pQCD, some nonperturbative models have been put forward and considered to be solutions to the problem, such as the intermediate meson exchange model [20] and the charmonium light Fock component admixture model [21], to further investigate it in pQCD is still necessary.

The rest of this paper is organized as follows: in section II, we calculate the branching ratio of the process $\eta_b \rightarrow J/\psi J/\psi$ at one-loop level; in section III, we evaluate the higher twist effects in processes $\eta_b \rightarrow J/\psi J/\psi$ and $\eta_c \rightarrow VV$; in section IV, summary and conclusions are presented.

II. NLO QCD RESULT FOR $\eta_b \rightarrow J/\psi + J/\psi$ PROCESS

In this section, we calculate the branching ratio of the process $\eta_b \rightarrow J/\psi + J/\psi$ in the framework of NRQCD at one-loop level and in non-relativistic limit. Hence, the relations $p_b = p_{\bar{b}} = \frac{P_{\eta_b}}{2}$, $p_{c_1} = p_{\bar{c}_1} = \frac{P_{J/\psi_1}}{2}$ and $p_{c_2} = p_{\bar{c}_2} = \frac{P_{J/\psi_2}}{2}$ are adopted. The bi-spinor operators are projected to states with the same quantum numbers as η_b and η_c , respectively, like

$$v(p_{\bar{b}}) \bar{u}(p_b) \longrightarrow \frac{-1}{2\sqrt{2} m_b} \left(\frac{P_{\eta_b}}{2} + m_b \right) \gamma_5 \left(\frac{P_{\eta_b}}{2} - m_b \right) \otimes \left(\frac{\mathbf{1}_c}{\sqrt{N_c}} \right), \quad (1)$$

and

$$v(p_{\bar{c}}) \bar{u}(p_c) \longrightarrow \frac{-1}{2\sqrt{2} m_c} \left(\frac{P_{J/\psi}}{2} - m_c \right) \not{\epsilon}^* \left(\frac{P_{J/\psi}}{2} + m_c \right) \otimes \left(\frac{\mathbf{1}_c}{\sqrt{N_c}} \right), \quad (2)$$

where $N_c = 3$, and $\mathbf{1}_c$ stands for the unit color matrix. In above, $M_{\eta_b} = 2m_b$ and $M_{J/\psi} = 2m_c$ are implicitly assumed.

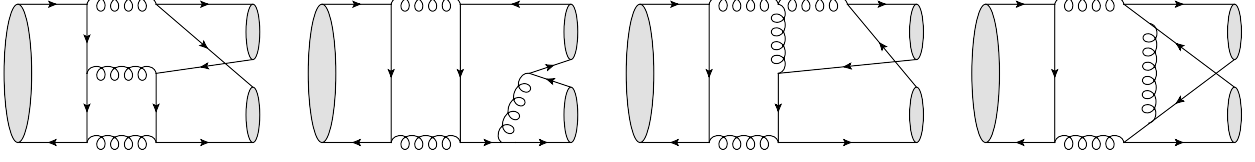


FIG. 1: Typical Feynman diagrams of the exclusive process $\eta_b(P_{\eta_b}) \rightarrow J/\psi(P_{J/\psi_1}) + J/\psi(P_{J/\psi_2})$ at the one-loop level.

For this process, at the leading order of relative velocity v in the framework of NRQCD, the tree level feynman-diagram has no contribution to the branching ratio, since the trace of the b-quark line form a Lorentz antisymmetric tensor, while the trace of c-quark line form a Lorentz symmetric tensor. This situation remains also in the NLO counterterm, self-energy and vertex correction diagrams. Therefore, at one-loop level only a few types of Feynman diagrams should be taken into account in the calculation, which are schemetically showed in Figure 1.

Because of parity and Lorentz invariance, the decay amplitude possesses the following unique tensor structure:

$$\mathcal{M}(\lambda_1, \lambda_2) = \mathcal{A} \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{J/\psi_1}^{*\mu}(\lambda_1) \varepsilon_{J/\psi_2}^{*\nu}(\lambda_2) P_{J/\psi_1}^\rho P_{J/\psi_2}^\sigma. \quad (3)$$

In our calculation, the computer algebra system MATHEMATICA is employed with the help of the packages, FYENCALC [22], FYENART [23], and LoopTools [24]. FYENART is used to draw Feynman diagrams and generate amplitudes, FYENCALC is used to trace the γ matrices and to reduce various Passarino-Veltman tensor integrals [25] to scalar ones, LoopTools is used to evaluate these integrals. For the aim of comparison, we express all the Passarino-Veltman scalar integrals encountered in this calculation in the Appendix.

After taking the above mentioned procedures, it is straightforward to calculate this process and obtain the analytical amplitude in reduced form. i.e.,

$$\mathcal{A} = \frac{512\sqrt{2}\pi\alpha_s^3 m_c \psi_{\eta_b}(0) \psi_{J/\psi}^2(0)}{9\sqrt{3}m_b^{9/2}(m_b^2 - 4m_c^2)} F(m_c^2, m_b^2), \quad (4)$$

where

$$F(m_c^2, m_b^2) = -\frac{1}{4}D_0(2)m_b^4 + 2D_0(1)m_c^2 m_b^2 + D_0(2)m_c^2 m_b^2 + \frac{19}{16}C_0(1)m_b^2 + \frac{3}{2}C_0(2)m_b^2 \\ - \frac{9}{4}C_0(3)m_b^2 - \frac{1}{4}C_0(5)m_b^2 - \frac{1}{4}C_0(6)m_b^2 - \frac{9}{16}C_0(7)m_b^2 - \frac{9}{4}C_0(1)m_c^2$$

$$\begin{aligned}
& -\frac{7}{4}C_0(2)m_c^2 - 2C_0(4)m_c^2 + \frac{9B_0(1)}{8} - \frac{9B_0(2)}{4} + 2B_0(3) - \frac{7B_0(4)}{8} \\
& + \frac{9B_0(5)}{8} - \frac{9B_0(6)}{8} + \frac{B_0(3)m_c^2}{m_b^2} - \frac{B_0(6)m_c^2}{m_b^2}.
\end{aligned} \tag{5}$$

Here, the form factor $F(m_c^2, m_b^2)$ is a complex function; D_0 , C_0 , and B_0 represent four-point, three-point and two-point Passarino-Veltman scalar integrals, respectively. The real part of $F(m_c^2, m_b^2)$ is too complicated to be presented here, and therefore only the asymptotic form in small m_c limit is given:

$$\begin{aligned}
\text{Re}(F(m_c^2, m_b^2))_{asy} = & \frac{19}{32}\log^2(a) - \frac{1}{8}\log(2)\log(a) + \frac{5}{4}\log(a) + \frac{5}{16}\log^2(2) \\
& + \frac{1}{2}\log(2) + \frac{29\pi^2}{96} - \frac{3\sqrt{3}}{8}\pi + \frac{3}{4}
\end{aligned} \tag{6}$$

with $a = \frac{m_c^2}{m_b^2}$. The full imaginary part of $F(m_c^2, m_b^2)$ is

$$\begin{aligned}
\text{Im}(F(m_c^2, m_b^2)) = & \frac{(36a - 19)\pi}{16\delta}\log\left(\frac{1 + \delta}{1 - \delta}\right) - \frac{(36a - 5)\pi}{16\delta}\log\left(\frac{3 - \delta}{3 + \delta}\right) \\
& + \frac{\pi}{4\delta}\log((1 + \delta)^2(1 + \delta^2)) + \frac{(2\delta^2 + 7\delta + 7)\pi}{8(\delta + 1)},
\end{aligned} \tag{7}$$

where $\delta = \sqrt{1 - 4a}$, and its asymptotic form in the small m_c limit reads

$$\text{Im}(F(m_c^2, m_b^2))_{asy} = \frac{19\pi}{16}\log(a) + \frac{7\pi}{16}\log(2) + \pi. \tag{8}$$

With the above preparation, we can readily obtain the branching fraction of the exclusive $\eta_b \rightarrow J/\psi J/\psi$ decay process,

$$Br[\eta_b \rightarrow J/\psi J/\psi] = K_{gg}^{-1} \frac{2^{13}\alpha_s^4 m_c^2 \psi_{J/\psi}^4(0)}{3^4 m_b^7 \sqrt{m_b^2 - 4m_c^2}} |F(m_c^2, m_b^2)|^2. \tag{9}$$

Here, the dominant η_b gluonic decay width is taken to be its total width approximately at one-loop order [26], i.e.,

$$\Gamma[\eta_b]_{total} \approx \Gamma_{NLO}[\eta_b \rightarrow gg] = K_{gg} \frac{8\pi\alpha_s^2}{3m_b^2} \psi_{\eta_b}^2(0) \tag{10}$$

with

$$K_{gg} = 1 + (C_F(-5 + \frac{\pi^2}{4}) + C_A(\frac{199}{18} - \frac{13\pi^2}{24}) - \frac{16}{9}n_f T_F) \frac{\alpha_s(2m_b)}{\pi}. \tag{11}$$

In numerical calculation, the following inputs are adopted:

$$\psi_{J/\psi}(0) = 0.263 \text{ GeV}^{3/2}, \quad m_c = 1.5 \text{ GeV}, \quad m_b = 4.7 \text{ GeV}, \quad \alpha_s = 0.18 \sim 0.26, \quad (12)$$

where the radial wave function at the origin $\psi_{J/\psi}(0)$ is obtained by fitting the NLO QCD calculation result to the J/ψ di-lepton decay width [27]. With the above preparation, we can readily obtain the numerical result of the concerned process

$$Br[\eta_b \rightarrow J/\psi J/\psi] = 5.93 \times 10^{-8} \sim 2.58 \times 10^{-7}. \quad (13)$$

Here, the uncertainties are originated from energy scale variation from m_b to m_c . It is worth emphasizing that in our result the double logarithms exist and agree with what obtained in Ref. [8], whereas our constant term does not agree with theirs, though its numerical influence is no big.

III. HIGHER TWIST CONTRIBUTIONS

As mentioned in preceding sections, at Born level $\Gamma(\eta_b \rightarrow J/\psi J/\psi)$ is exactly zero in non-relativistic limit, while the NLO radiative corrections are very small. People find that although in light cone formalism the leading twist term in the light cone distribution amplitudes(LCDAs) for J/ψ vanishes in $\eta_b \rightarrow J/\psi J/\psi$ process, contributions from higher twist terms seem to be important [9]. In Ref. [9] the LCDAs up to twist-4 are taken into account for the consistency reason. It is true and in the following we reevaluate this process also in the light cone framework. However, to execute the twist expansion accurately, for final vector mesons with transverse polarizations, we expand the LCDA projector in momentum space given by [28] to twist-4, which yields more terms than what employed in Ref. [9]. i.e.,

$$M_\perp^V = (M_\perp^{(2)} + M_\perp^{(3)} + M_\perp^{(4)}) \Big|_{k=up}, \quad (14)$$

where

$$M_\perp^{(2)} = \frac{1}{4} f_V^T E \not{\epsilon}_\perp \not{p}_- \phi_\perp(u), \quad (15)$$

$$\begin{aligned} M_\perp^{(3)} = & \frac{1}{4} f_V m_V \left[\not{\epsilon}_\perp g_\perp^{(v)}(u) - E \not{p}_- \int_0^u dv \left(\phi_\parallel(v) - g_\perp^{(v)}(v) \right) \varepsilon_\perp^\sigma \frac{\partial}{\partial k_\perp^\sigma} \right] \\ & + \frac{i}{4} \left(f_V - f_V^T \frac{m_1 + m_2}{m_V} \right) m_V \varepsilon_{\mu\nu\rho\sigma} \varepsilon_\perp^\nu n_-^\rho \gamma^\mu \gamma_5 \left(n_+^\sigma \frac{g_\perp^{(a)}(u)}{8} - E \frac{g_\perp^{(a)}(u)}{4} \frac{\partial}{\partial k_{\perp\sigma}} \right), \quad (16) \end{aligned}$$

$$\begin{aligned}
M_{\perp}^{(4)} = & \frac{1}{4} \frac{m_V^2}{E} f_V^T \left[\frac{1}{4} \not{\epsilon}_{\perp} \not{k}_+ h_3(u) - \frac{E}{4} [\not{k}_-, \not{k}_+] \int_0^u dv \left(h_{\parallel}^{(t)}(v) - \frac{1}{2} \phi_{\perp}(v) - \frac{1}{2} h_3(v) \right) \varepsilon_{\perp}^{\sigma} \frac{\partial}{\partial k_{\perp}^{\sigma}} \right] \\
& - \frac{1}{4} m_V^2 \left(f_V^T - f_V \frac{m_1 + m_2}{m_V} \right) \frac{h_{\parallel}^{(s)}(u)}{2} \varepsilon_{\perp}^{\sigma} \frac{\partial}{\partial k_{\perp}^{\sigma}}, \tag{17}
\end{aligned}$$

with $E = (p^0 + |\vec{p}|)/2$ and the transverse polarization vector

$$\epsilon_{\perp}^{\mu} = \varepsilon^{\mu} - \frac{\varepsilon \cdot n_+}{2} n_+^{\mu} - \frac{\varepsilon \cdot n_-}{2} n_-^{\mu}. \tag{18}$$

The higher twist LCDAs are related to the twist-2 ones by the Wandzura-Wilczek relations[30],

$$g_{\perp}^{(v)}(u) = \frac{1}{2} \left[\int_0^u \frac{\phi_{\parallel}(v)}{\bar{v}} dv + \int_u^1 \frac{\phi_{\parallel}(v)}{v} dv \right], \tag{19}$$

$$g_{\perp}^{(a)}(u) = 2 \left[\bar{u} \int_0^u \frac{\phi_{\parallel}(v)}{\bar{v}} dv + u \int_u^1 \frac{\phi_{\parallel}(v)}{v} dv \right], \tag{20}$$

$$h_{\parallel}^{(s)}(u) = 2 \left[\bar{u} \int_0^u \frac{\phi_{\perp}(v)}{\bar{v}} dv + u \int_u^1 \frac{\phi_{\perp}(v)}{v} dv \right]. \tag{21}$$

Here, the contributions from three-particle DAs have been neglected as performed in Ref. [9].

After a lengthy calculation, we get the final expression for the decay amplitude of the process $\eta_Q \rightarrow V_1 V_2$,

$$\begin{aligned}
\mathcal{M}_{\perp\perp} = & T_0 \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{1\perp}^{*\mu} \varepsilon_{2\perp}^{*\nu} n_-^{\rho} n_+^{\sigma} \int_0^1 du_1 \int_0^1 d\bar{u}_2 \frac{1}{256 E_1^2 E_2^2 u_1 \bar{u}_1 u_2 \bar{u}_2 (u_1 u_2 + \bar{u}_1 \bar{u}_2)} \\
& \times \left\{ m_{V_1} m_{V_2} f_{V_1} \tilde{f}_{V_2} \left[g_{1\perp}^{(v)}(u_1) g_{2\perp}^{(a)}(\bar{u}_2) + \Phi_1(u_1) g_{2\perp}^{'(a)}(\bar{u}_2) + f(u_1, \bar{u}_2) \Phi_1(u_1) g_{2\perp}^{(a)}(\bar{u}_2) \right] \right. \\
& + m_{V_1} m_{V_2} f_{V_2} \tilde{f}_{V_1} \left[g_{2\perp}^{(v)}(\bar{u}_2) g_{1\perp}^{(a)}(u_1) + \Phi_2(\bar{u}_2) g_{1\perp}^{'(a)}(u_1) - f(u_1, \bar{u}_2) \Phi_2(\bar{u}_2) g_{1\perp}^{(a)}(u_1) \right] \\
& \left. + 2m_{V_2}^2 f_{V_1}^T \tilde{f}_{V_2}^T \phi_{1\perp}(u_1) h_{2\parallel}^{(s)}(\bar{u}_2) + 2m_{V_1}^2 f_{V_2}^T \tilde{f}_{V_1}^T \phi_{2\perp}(\bar{u}_2) h_{1\parallel}^{(s)}(u_1) \right\}, \tag{22}
\end{aligned}$$

with

$$T_0 = 8g_s^4 \frac{\psi_{\eta_Q}(0)}{\sqrt{8m_Q}} \frac{N_c^2 - 1}{4N_c^2 \sqrt{N_c}}, \tag{23}$$

$$f(u_1, \bar{u}_2) = (u_1 - \bar{u}_2) \left(\frac{-1}{u_1 \bar{u}_2} + \frac{-1}{u_2 \bar{u}_1} + \frac{2}{u_1 u_2 + \bar{u}_1 \bar{u}_2} \right), \tag{24}$$

$$\tilde{f}_V = f_V - f_V^T \frac{m_1 + m_2}{m_V}, \quad \tilde{f}_V^T = f_V^T - f_V \frac{m_1 + m_2}{m_V}, \tag{25}$$

$$\Phi_1(u_1) = \int_0^{u_1} dw \left(\phi_{1\parallel}(w) - g_{1\perp}^{(v)}(w) \right), \quad \Phi_2(\bar{u}_2) = \int_0^{\bar{u}_2} dw \left(\phi_{2\parallel}(w) - g_{2\perp}^{(v)}(w) \right). \tag{26}$$

Here, $\psi_{\eta_Q}(0)$ is the wave function at the origin for pseudoscalar η_Q . Our analytical result is different from what given in Ref.[9], partly due to the different projectors used.

With the asymptotic form for twist-2 distribution amplitudes,

$$\phi_{\perp}(u) = \phi_{\parallel}(u) = \phi_{AS}(u) = 6u(1-u) , \quad (27)$$

the analytical decay amplitude turns to be pretty simple, it reads

$$\begin{aligned} \mathcal{M}_{\perp\perp} = & T_0 \varepsilon_{\mu\nu\rho\sigma} \varepsilon_{1\perp}^{*\mu} \varepsilon_{2\perp}^{*\nu} n_-^{\rho} n_+^{\sigma} \frac{9}{256 E_1^2 E_2^2} \times \left[(\pi^2 - 4) m_{V_1} m_{V_2} (f_{V_1} \tilde{f}_{V_2} + f_{V_1} \tilde{f}_{V_2}) \right. \\ & \left. + 2\pi^2 (m_{V_2}^2 f_{V_1}^T \tilde{f}_{V_2}^T + m_{V_1}^2 f_{V_2}^T \tilde{f}_{V_1}^T) \right] . \end{aligned} \quad (28)$$

To numerically evaluate the branching ratio of $\eta_b \rightarrow J/\psi J/\psi$ process, we use the following commonly accepted input parameters: the charm quark mass in the \overline{MS} scheme, $m_c^{\overline{MS}} = 1.2$ GeV; the J/ψ decay constant $f_{J/\psi} = 416$ MeV; and the $f_{J/\psi}^T$ is obtained in the framework of NRQCD [29], $f_{J/\psi}^T = 379$ MeV. Then the numerical result reads

$$Br[\eta_b \rightarrow J/\psi J/\psi] = (1.1 \sim 2.3) \times 10^{-6} . \quad (29)$$

Here, the uncertainties are also induced by the scale variation from m_b to m_c as in above NLO QCD calculation. Note that the above magnitude is bigger than what the NLO result. This is however understandable, since roughly speaking the higher twist contributions are suppressed by factor of $\left(\frac{m_c}{m_b}\right)^4$ while the NLO contributions are suppressed by $\alpha_s^2 \left(\frac{m_c}{m_b}\right)^2$.

Following we apply the above higher twist analysis to the η_c to light vector mesons decay process for the first time to twist-4. As mentioned in the introduction the disagreement of experimental measurement with theoretical expectation is a long lasting issue. Before attributing some non-perturbative scenarios, to evaluate these processes in light cone formalism to twist-4 is still meaningful. We already know that the leading twist term in LCDAs at leading order of α_s does not contribute to these processes, and the radiative corrections are dramatically suppressed by the light quark masses. Therefore, it is obvious that contributions from higher twist DAs dominate over others in the framework of perturbative QCD. In addition to the asymptotic form, the LCDA form in terms of Gegenbauer polynomials is also employed. That is

$$\phi_{\parallel,\perp}(u, \mu^2) = 6u(1-u) \left(1 + \sum_{n=1}^{\infty} a_n^{\parallel,\perp}(\mu^2) C_n^{3/2}(2u-1) \right) . \quad (30)$$

The input parameters needed in the numerical calculation are listed in TABLE. I, f_V^T s come from the QCD sum rules [30–33], and the reasonable values of Gegenbauer moments a_1 and a_2 are from Ref. [34]. All scale-dependent quantities refer to $\mu = 1$ GeV. The numerical results

TABLE I: Summary of theoretical input parameters.

	ρ	\bar{K}^*	ω	ϕ
$m_V[\text{MeV}]$	770	892	782	1020
$f_V[\text{MeV}]$	205 ± 9	217 ± 5	195 ± 3	231 ± 4
$f_V^T[\text{MeV}]$	160 ± 10	170 ± 10	145 ± 10	200 ± 10
$a_1^{\parallel,\perp}$	0	0.10 ± 0.07	0	0
$a_2^{\parallel,\perp}$	$0.09^{+0.10}_{-0.07}$	$0.07^{+0.09}_{-0.07}$	$0.09^{+0.10}_{-0.07}$	$0.06^{+0.09}_{-0.07}$

are given in Table. II, where Br[AS] and Br[GP] represent the results for forms of asymptotic and the Gegenbauer polynomials in LCDAs, respectively. Since the Gegenbauer moments a_n are small, the numerical results are not sensitive to the form of the leading distribution amplitudes, which influence the higher twist results via relations (19)-(21). From results in Table II we see that although the higher twist effect are tremendous for Br[$\eta_c \rightarrow VV$], it is still not enough to explain the experimental data.

TABLE II: Experimental data and Numerical results for $Br[\eta_c \rightarrow VV]$, experimental data are from Particle Data Book [27].

Final state	Br[ex]	Br[AS]	Br[GP]
$\rho\rho$	$(2.0 \pm 0.7) \times 10^{-2}$	2.0×10^{-4}	2.8×10^{-4}
$K^*\bar{K}^*$	$(9.2 \pm 3.4) \times 10^{-3}$	7.2×10^{-4}	9.0×10^{-4}
$\omega\omega$	$< 3.1 \times 10^{-3}$	9.1×10^{-5}	1.3×10^{-4}
$\phi\phi$	$(2.7 \pm 0.9) \times 10^{-3}$	6.6×10^{-4}	8.1×10^{-4}

IV. CONCLUSIONS

To further study the nature of recently observed state η_b , direct measurement of its decay products is necessary. The $\eta_b \rightarrow J/\psi J/\psi$ process was considered and suggested to be a golden channel to this aim. In the literature, different theoretical estimation varies

greatly. The branching ratio starts from 10^{-4} to 10^{-8} , which induces some confusion for future experimental test. Within the pQCD and factorization scheme we have calculated this helicity conservation suppressed process, the $\eta_b \rightarrow J/\psi J/\psi$, at the next-to-leading order in pQCD. Our result confirms the existence of double logarithms, the $\log^2(\frac{m_c^2}{m_b^2})$ in Ref. [8], and the coefficients of both double logarithm $\log^2(\frac{m_c^2}{m_b^2})$ and single logarithm $\log(\frac{m_c^2}{m_b^2})$ in our calculation are consistent with those in the same reference. However, we find that other terms in our result deviate from those in Ref. [8], though the numerical significance of the difference is not high.

In the light cone formalism, the leading twist contribution to $\eta_b \rightarrow J/\psi J/\psi$ process vanishes. In this work we also evaluate the higher twist contributions to it. Expanding the LCDAs of final vector mesons to twist-4, we find that the higher twist terms contribute more to the decay width than what from the NLO corrections, which implies that the final state mass effects is more significant than the NLO corrections in this helicity suppressed process. According to our twist-4 calculation, the branching fraction of $\eta_b \rightarrow J/\psi J/\psi$ process can be as large as $\sim 10^{-6}$, which enables the direct search of η_b in Tevatron Run II or LHC feasible. In Ref. [9], the same process was evaluated in the light cone formalism also to twist-4, but with different twist expansion procedure, which lead to different LCDAs from ours. We believe what we used are generated from the LCDA definition in twist expansion and should be more proper.

Unlike the undiscovered $\eta_b \rightarrow J/\psi J/\psi$ process, experimental results about $\eta_c \rightarrow VV$ indicate that relatively large violations of the helicity conservation rules exist in these processes. The surprisingly large branching ratios of $\eta_c \rightarrow VV$ still stand as a bewildering puzzle in charmonium physics. Since it is believed that the NLO corrections are greatly suppressed by the light quark mass, the higher twist effects might be large. In the light cone formalism, we have calculated this process with taking the next-to-next-leading twist effects in LCDAs of final vector mesons into account. Result shows that the higher twist DAs indeed violate the helicity conservation rules, though it still deviates a lot from the experimental measurement. This implies that the perturbative description of η_c decay alone is not enough, and some non-perturbative mechanism may play important roles in η_c decays, such as $\eta_c \rightarrow VV$, which deserves further investigation [35].

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Appendix

In this appendix, we list various Passarino-Veltman scalar integrals appearing in Eq.(5), and only the leading power in a for real part are extracted; while for imaginary part, we present the full expression. Since coefficients of the $D_0(1)$ and $C_0(4)/m_b^2$ in Eq.(5) are same up to a sign, hence we only present their difference. Our evaluation for these integrals agrees with LoopTools, and we have also checked with FIESTA2 [36] the asymptotic expression. Here, $a = \frac{m_c^2}{m_b^2}$ and $\delta = \sqrt{1 - 4a}$.

$$\begin{aligned} D_0(1) &= D_0[m_b^2, m_c^2, 4m_c^2, m_c^2, m_c^2, 2m_b^2 + m_c^2, 0, 0, m_c^2, m_c^2] \\ &= \frac{C_0(4)}{m_b^2} - \frac{\log(2)}{m_c^2 m_b^2} \end{aligned} \quad (31)$$

$$\begin{aligned} D_0(2) &= D_0[m_b^2, m_c^2, 4m_b^2, m_c^2, 2m_b^2 + m_c^2, 2m_b^2 + m_c^2, m_c^2, m_c^2, 0, 0] \\ &\approx \frac{1 - \log(a)}{2m_b^4} - i\pi \frac{1}{4m_b^2(1 + \delta)} \end{aligned} \quad (32)$$

$$\begin{aligned} C_0(1) &= C_0[m_b^2, m_c^2, m_c^2, 0, 0, m_c^2] \\ &\approx \frac{3\log^2(a) + \pi^2}{6m_b^2} - i\pi \frac{\log(\frac{1+\delta}{1-\delta})}{m_b^2 \delta} \end{aligned} \quad (33)$$

$$\begin{aligned} C_0(2) &= C_0[m_b^2, m_c^2, 2m_b^2 + m_c^2, 0, 0, m_c^2] \\ &\approx \frac{-6\log(2)\log(a) + 3\log^2(2) + \pi^2}{6m_b^2} - i\pi \frac{\log(\frac{3-\delta}{3+\delta})}{m_b^2 \delta} \end{aligned} \quad (34)$$

$$\begin{aligned} C_0(3) &= C_0[4m_b^2, m_c^2, 2m_b^2 + m_c^2, 0, 0, m_c^2] \\ &\approx \frac{-6\log(2)\log(a) + 3\log^2(2) + \pi^2}{12m_b^2} - i\pi \frac{\log(\frac{3-\delta}{3+\delta})}{2m_b^2 \delta} \end{aligned} \quad (35)$$

$$C_0(4) = C_0[m_c^2, m_c^2, 4m_c^2, m_c^2, 0, m_c^2] \quad (36)$$

$$\begin{aligned} C_0(5) &= C_0[m_b^2, m_c^2, m_c^2, m_c^2, m_c^2, m_b^2] \\ &\approx -\frac{\pi^2}{12m_b^2} - \mathrm{i}\pi \frac{\log(2-4a)}{m_b^2\delta} \end{aligned} \quad (37)$$

$$\begin{aligned} C_0(6) &= C_0[m_b^2, m_c^2, 2m_b^2 + m_c^2, m_c^2, m_c^2, 0] \\ &\approx \frac{-6\log(2)\log(a) - 3\log^2(2) + \pi^2}{6m_b^2} - \mathrm{i}\pi \frac{\log(\frac{(3-\delta)(1+\delta)^2}{3+\delta})}{m_b^2\delta} \end{aligned} \quad (38)$$

$$C_0(7) = C_0[m_b^2, m_c^2, m_c^2, m_b^2, m_b^2, m_c^2] \approx -\frac{\pi^2}{9m_b^2} \quad (39)$$

$$B_0(1) = B_0[m_b^2, 0, 0] \approx \frac{1}{\epsilon} + 2 - \log(m_b^2) + \mathrm{i}\pi \quad (40)$$

$$B_0(2) = B_0[4m_b^2, 0, 0] = \frac{1}{\epsilon} + 2 - \log(4m_b^2) + \mathrm{i}\pi \quad (41)$$

$$B_0(3) = B_0[2m_b^2 + m_c^2, 0, m_c^2] \approx \frac{1}{\epsilon} + 2 - \log(2m_b^2) + \mathrm{i}\pi \frac{2}{2+a} \quad (42)$$

$$B_0(4) = B_0[m_c^2, m_b^2, m_c^2] \approx \frac{1}{\epsilon} + 1 - \log(m_b^2) \quad (43)$$

$$B_0(5) = B_0[m_b^2, m_b^2, m_b^2] = \frac{1}{\epsilon} + 2 - \frac{\pi}{\sqrt{3}} - \log(m_b^2) \quad (44)$$

$$B_0(6) = B_0[m_c^2, 0, m_c^2] = \frac{1}{\epsilon} + 2 - \log(m_c^2) \quad (45)$$

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